New Specification

Rewarding Learning

General Certificate of Secondary Education
2018

Further Mathematics

Unit 1 **(With calculator)**
Pure Mathematics

[GFM11]

TUESDAY 12 JUNE, MORNING

MARK SCHEME

General Marking Instructions

Introduction

Mark schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

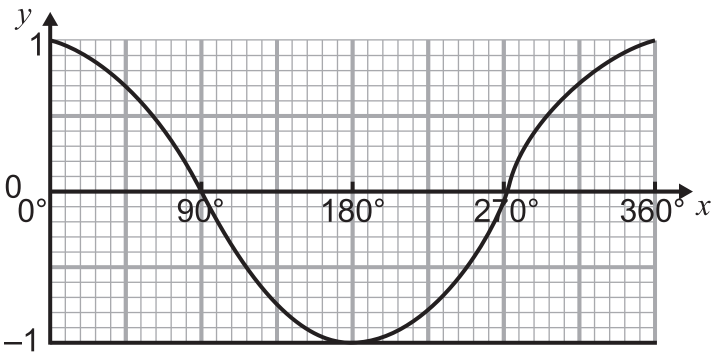
Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of students in schools and colleges.

The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes, therefore, are regarded as part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

			AVAILABLE MARKS
1 **(i)** $y = \frac{1}{4}x^5 - \frac{6}{x^2} + 10$			
$= \frac{1}{4}x^5 - 6x^{-2} + 10$			
$\frac{dy}{dx} = \frac{5}{4}x^4 + 12x^{-3}$ or $\frac{5}{4}x^4 + \frac{12}{x^3}$		3 × MW1	
(ii) $\frac{d^2y}{dx^2} = 5x^3 - 36x^{-4}$ or $5x^3 - \frac{36}{x^4}$		2 × MW1	5
2 $\int_1^2 \left(3x^2 - \frac{2}{x^2} + 1\right)dx$			
$= \int_1^2 (3x^2 - 2x^{-2} + 1)\, dx$			
$= \left[x^3 + \frac{2}{x} + x\right]_1^2$		3 × MW1	
$= [8 + 1 + 2] - [1 + 2 + 1]$		M1	
$= 11 - 4$			
$= 7$		W1	5
3 **(a)** 		M1 (accuracy) W1 (curve)	
(b) $\cos\left(\frac{4}{5}x + 30°\right) = -0.5$			
$\frac{4}{5}x + 30° = 120°$ or	$240°$	MW1 MW1	
$\frac{4}{5}x + 30° = 120°$ or	$\frac{4}{5}x + 30° = 240°$	M1	
$\frac{4}{5}x = 90°$ or	$\frac{4}{5}x = 210°$		
$x = 112.5°$ or	$x = 262.5°$	W1 W1	7

AVAILABLE MARKS

4 $2x^2 + 3x - 4 \leqslant 5$

$2x^2 + 3x - 9 \leqslant 0$ MW1

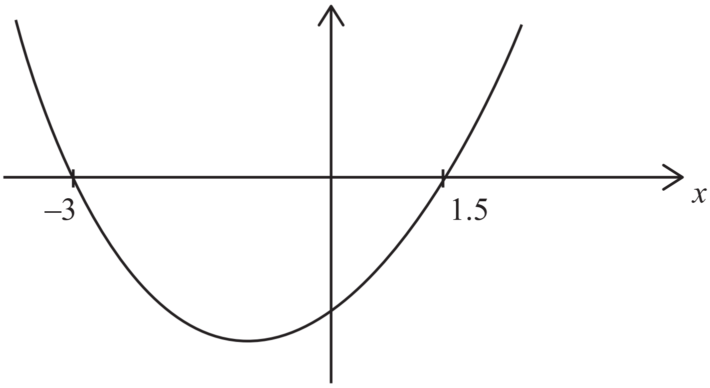
Find roots

$2x^2 + 3x - 9 = 0$

$(2x - 3)(x + 3) = 0$ M1

$x = 1.5$ or $x = -3$ W1

Sketch:



Alternatively:

		–3	1.5
$2x - 3$	–	–	+
$x + 3$	–	+	+
$(2x - 3)(x + 3)$	+	–	+

$-3 \leqslant x \leqslant 1.5$ W1 W1 5

5 Let $\mathbf{A} = \mathbf{P} - \mathbf{Q}$

$$\mathbf{A} = \begin{bmatrix} 3 & -1 \\ 2 & 8 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$$ M1 W1

$\mathbf{AX} = \mathbf{R}$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{R}$

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix}$$ M1 MW1 MW1

$$\mathbf{X} = \frac{1}{3}\begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$ M1

$$= \frac{1}{3}\begin{bmatrix} 15 \\ -18 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$ W1 7

AVAILABLE MARKS

6 $3^{2x-1} = 7^{x+2}$

$\log 3^{2x-1} = \log 7^{x+2}$ M1

$(2x - 1) \log 3 = (x + 2) \log 7$ MW1

$2x \log 3 - \log 3 = x \log 7 + 2 \log 7$ W1

$2x \log 3 - x \log 7 = 2 \log 7 + \log 3$

$x (2 \log 3 - \log 7) = 2 \log 7 + \log 3$ M1

$x = \dfrac{2 \log 7 + \log 3}{2 \log 3 - \log 7}$

$x = 19.857 \rightarrow 19.86$ W1

Alternative Solution

$2x - 1 = \log_3 7^{x+2}$ M1

$2x - 1 = (x + 2) \log_3 7$ MW1

$2x - x \log_3 7 = 1 + 2 \log_3 7$ W1

$x(2 - \log_3 7) = 1 + 2 \log_3 7$

$x = \dfrac{1 + 2 \log_3 7}{2 - \log_3 7}$ M1

$= \dfrac{1 + 2\,(1.771243749)}{2 - 1.771243749}$

$= 19.86$ W1

(or similarly if $\log_7 3^{2x-1} = x + 2$) 5

			AVAILABLE MARKS
7	$y = \frac{5}{2}x^2 + 6 - \frac{40}{x}$		
	$= \frac{5}{2}x^2 + 6 - 40x^{-1}$		
	$\frac{dy}{dx} = 5x + 40x^{-2}$ or $5x + \frac{40}{x^2}$	MW1 MW1	
	Tangent is horizontal, so $\frac{dy}{dx} = 0$		
	$5x + \frac{40}{x^2} = 0$	M1	
	$5x = -\frac{40}{x^2}$		
	$5x^3 = -40$	MW1	
	$x^3 = -8$		
	$x = -2$	W1	
	$y = \frac{5}{2}(-2)^2 + 6 - \frac{40}{-2} = 36$		
	Point is (–2, 36)	W1	6
8	**(i)** $\frac{x^2 + 2x - 3}{x(x+3)} + \frac{2x}{x+1}$		
	$= \frac{(x+3)(x-1)}{x(x+3)} + \frac{2x}{x+1}$	MW1 (factors)	
	$= \frac{x-1}{x} + \frac{2x}{x+1}$	W1 (cancelling)	
	$= \frac{(x-1)(x+1) + 2x^2}{x(x+1)} = \frac{x^2 - 1 + 2x^2}{x(x+1)}$	MW1 (numerator) MW1 (denominator)	
	$= \frac{3x^2 - 1}{x(x+1)}$	W1	
	(ii) $\frac{x-1}{x} + \frac{2x}{x+1} = 2$		
	$\frac{3x^2 - 1}{x(x+1)} = 2$		
	$3x^2 - 1 = 2x(x+1)$	M1	
	$3x^2 - 1 = 2x^2 + 2x$	W1	
	$x^2 - 2x - 1 = 0$		
	(iii) $(x-1)^2 - 2 = 0$	MW1 $(x-1)^2$, MW1 for –2	
	$x = 1 \pm \sqrt{2}$	W1	10

9 **(i)** $x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$ M1

$x(x - 3)^2 = 0$ MW1

$x = 0$ or 3

Points are (0, 0), (3, 0) W1

(ii) $\dfrac{dy}{dx} = 3x^2 - 12x + 9$ M1 W1

$= 0$ M1

$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1$ or 3 W1
$y = 4$ or 0

Turning points at (1, 4) and (3, 0) W1

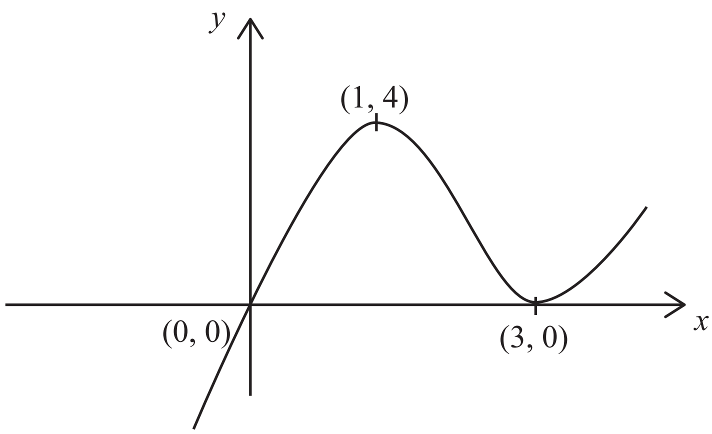
(iii) $\dfrac{d^2y}{dx^2} = 6x - 12$ MW1

When $x = 1$, $\dfrac{d^2y}{dx^2} = -6 < 0 \quad \therefore$ max

When $x = 3$, $\dfrac{d^2y}{dx^2} = 6 > 0 \quad \therefore$ min

Minimum at (3, 0)
Maximum at (1, 4) MW1

(iv)



M1 (shape)
W1 (points)

AVAILABLE MARKS

12

10	**(i)**	$5x + 3y + 2z = 1985$	W1
	(ii)	$z = x + y + 55$ $-x - y + z = 55$	W1
	(iii)	$7(x - 30) + 5(y - 35) + z = 1840$	M1
		$7x - 210 + 5y - 175 + z = 1840$	W1
		$7x + 5y + z = 2225$	
	(iv)	$5x + 3y + 2z = 1985$ (1)	
		$-x - y + z = 55$ (2)	
		$7x + 5y + z = 2225$ (3)	
		$5x + 3y + 2z = 1985$ (1)	
		$-2x - 2y + 2z = 110$ (2) × 2	
		$7x + 5y = 1875$ (4)	M1 W1
		$7x + 5y + z = 2225$ (3)	
		$-x - y + z = 55$ (2)	
		$8x + 6y = 2170$ (5)	M1 W1
		$42x + 30y = 11\,250$ (4) × 6	
		$40x + 30y = 10\,850$ (5) × 5	
		$2x = 400$	
		$x = 200$	M1 W1
		Back substitution $y = 95$, $z = 350$	M1
		Mountain bike £200 Child's bike £95 Road bike £350	W1
	(v)	£980 ÷ 4 = £245 £350 –£245 = £105 reduction $\frac{105}{350} \times 100 = 30\%$	M1 W1

Alternative solution

Original cost of 4 bikes = 350 × 4 = £1400
Reduction = 1400 – 980 = 420
Percentage reduction = $\frac{420}{1400} \times 100 = 30\%$ M1 W1

AVAILABLE MARKS: 14

11 **(i)** $T = An^k$

$\log T = \log An^k = \log A + \log n^k$

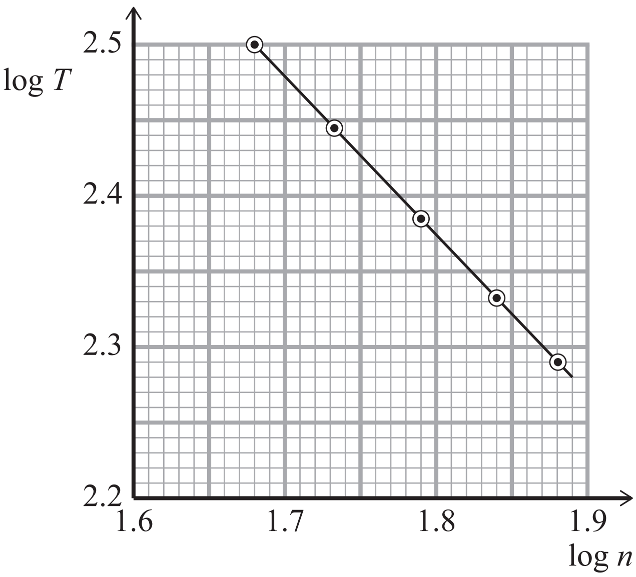
$\log T = k \log n + \log A$ M1

Marathon	Training runs, *n*	Time taken, *T*	log *n*	log *T*
1	48	315.44	1.681	2.499
2	54	278.70	1.732	2.445
3	62	241.10	1.792	2.382
4	69	215.49	1.839	2.333
5	76	194.70	1.881	2.289

M1 (taking logs)

W1 (all log *n* answers correct to 3 dp)

W1 (all log *T* answers correct to 3 dp)



W1 (labels)

W1 (points plotted correctly)

W1 (straight line through points)

(ii) $k = \dfrac{2.289 - 2.499}{1.881 - 1.681} = \dfrac{-0.21}{0.2} = -1.05$ M1 W1

$An^k = T$

$A\,(48)^{-1.05} = 315.44$ M1

$0.017167A = 315.44$

$A = 18\,374.692 \rightarrow 18374.69$ W1

(iii) $T = An^k$

$T = 18374.69 \times 60^{-1.05}$

$T = 249.55$ minutes MW1

AVAILABLE MARKS

			AVAILABLE MARKS
(iv)	$18374.69\, n^{-1.05} = 185$		
	$n^{-1.05} = 0.010068197$		
	$n = (0.010068197)^{\frac{-1}{1.05}}$	M1	
	(or $-1.05 \log n = \log 0.010068197$)		
	$n = 79.79 \rightarrow 80$ training runs	W1	
	Assumption: The relationship holds outside the range	M1	15
12 (i)	$N = N_A + N_B$		
	$= 2000\left(1 - \frac{x^2}{100}\right) + 1000\left(1 - \frac{y^2}{200}\right)$	M1	
	$= 2000 - 20x^2 + 1000 - 5y^2$	W1	
	$= 3000 - 20x^2 - 5y^2$		
(ii)	$x + y = 8$ $y = 8 - x$	MW1	
	$N = 3000 - 20x^2 - 5(8 - x)^2$	M1	
	$= 3000 - 20x^2 - 5(64 - 16x + x^2)$		
	$= 3000 - 20x^2 - 320 + 80x - 5x^2$	W1	
	$= 2680 + 80x - 25x^2$		
(iii)	$\frac{dN}{dx} = 80 - 50x$	MW1	
	$= 0$ for max	M1	
	$x = \frac{80}{50} = 1.6$	W1	
	$\frac{d^2N}{dx^2} = -50 < 0 \therefore$ max	MW1	
	Store should be built 1.6 km from A		9
		Total	**100**